Equivalent Network for an Aperture in the Center Conductor of Microstrip Line

K. Srinivas Rao and V. M. Pandharipande

Abstract —**The equivalent circuits for discontinuities in the form of a thin transverse slot and a circular aperture in the center conductor of a microstrip line are determined by a quasi-static analysis using equivalent dipole moments. The series reactance as a function of frequency is evaluated in terms of the geometrical parameters of the discontinuity. Data obtained from transmission measurements are presented to support the theory.**

I. Introduction

In order to use a discontinuity in a planar transmission line as a circuit element, knowledge of the lumped equivalent network, in terms of the geometrical parameters of the discontinuity, is essential. Discontinuities in the ground planes of striplines and microstrip lines have been extensively investigated [1], [2]. The discontinuity in the form of apertures in the ground plane may radiate into free space (as in slot radiators) or may couple the electromagnetic energy into another transmission line (as in aperture couplers). The impedance characteristics of a slot in the ground plane of a stripline have been determined by Rao and Das [1]. Das and Joshi have evaluated the complex admittance of a radiating slot in the ground plane of a microstrip line from a knowledge of the complex radiated power and the discontinuity in modal voltage [2].

Step, gap, and slit discontinuities in the center conductor of microstrip have been investigated by various authors [3]–[5]. Oliner has derived formulas for the equivalent circuit of a discontinuity in the center conductor of a strip transmission line using a small-aperture procedure [6]. Hoefer has presented results for the series inductance of a transverse slit in a microstrip line using the concept of volume ratios and a parallel-plate model of the line with magnetic walls [5]. However, to the best of authors' knowledge, the problem of discontinuities in the form of a transverse slot or a circular aperture in the conducting strip of a microstrip line has not been addressed.

This paper presents results for the series reactance of discontinuities in the form of a thin transverse slot and a circular aperture in the center conductor of a microstrip line. The method uses a quasi-static analysis and makes use of the planar waveguide model of microstrip line and equivalent dipole moments. The results are compared with those obtained by modifying Oliner's formulas [7] and the complex power method of Das *et al.* [2]. Experimental data in the lower edge of the microwave band (0.5 to 2 GHz) are presented to support the theory.

II. Formulation and Method of Solution

Fig. 1(a) shows the thin slot of width t and length l in the center conductor of the microstrip line having width W. The electric and magnetic fields are expressed in terms of modal vectors $\boldsymbol{e}$ and $\boldsymbol{h}$ which satisfy the normalizing condition:

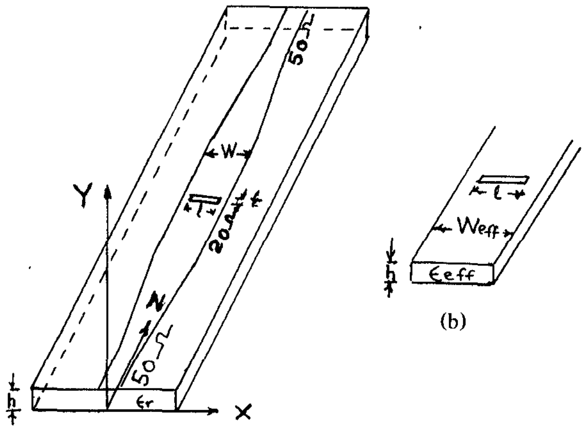

Fig. 1. Geometry of the discontinuity in microstrip line and its parallel-plate equivalent.

$$\iint \boldsymbol{e}\cdot\boldsymbol{e}\,dS = \iint \boldsymbol{h}\cdot\boldsymbol{h}\,dS = 1. \tag{1}$$

Fig. 1(b) shows the equivalent parallel-plate configuration where the microstrip line has been represented by a parallel-plate waveguide of width W_{eff}, with electric walls at the top and bottom and magnetic walls at the sides.

The normalized modal vectors for the TEM mode in the parallel-plate line are given by [8]

$$\boldsymbol{e} = \boldsymbol{a}_y\sqrt{1/W_{\text{eff}}\cdot h} \tag{2a}$$

$$\boldsymbol{h} = -\boldsymbol{a}_x\sqrt{1/W_{\text{eff}}\cdot h} \tag{2b}$$

where W_{eff}, the equivalent parallel-plate width, is given by [9]. Positive and negative traveling waves are given by

$$\boldsymbol{E}^+ = \boldsymbol{e}e^{-j\beta z} \qquad \boldsymbol{H}^+ = \boldsymbol{h}Y_0e^{-j\beta z} \tag{3a}$$

$$\boldsymbol{E}^- = \boldsymbol{e}e^{j\beta z} \qquad \boldsymbol{H}^- = -\boldsymbol{h}Y_0e^{j\beta z}. \tag{3b}$$

Let the incident mode be

$$\boldsymbol{E}_i = A_1\boldsymbol{E}^+ = A_1\boldsymbol{e}e^{-j\beta z} \tag{4a}$$

$$\boldsymbol{H}_i = A_1\boldsymbol{H}^+ = A_1\boldsymbol{h}Y_0e^{-j\beta z} \tag{4b}$$

where A_1 is the amplitude constant and

$$Z_0 = 1/Y_0 = 120\pi/\sqrt{\epsilon_{r\,\text{eff}}}. \tag{5}$$

Let the scattered field due to the discontinuity in the microstrip line be [10]

$$a_1\boldsymbol{E}^+, \qquad a_1\boldsymbol{H}^+ \qquad \text{for } z>0 \tag{6a}$$

$$b_1\boldsymbol{E}^-, \qquad b_1\boldsymbol{H}^- \qquad \text{for } z<0 \tag{6b}$$

due to electric dipole and

$$a_2\boldsymbol{E}^+, \qquad a_2\boldsymbol{H}^+ \qquad \text{for } z>0 \tag{7a}$$

$$b_2\boldsymbol{E}^-, \qquad b_2\boldsymbol{H}^- \qquad \text{for } z<0 \tag{7b}$$

due to magnetic dipole.

The coefficients a_1, a_2, b_1, and b_2 are given by [10]

$$a_1 = -j\omega\boldsymbol{E}^-\cdot\boldsymbol{P}_0/2P_n \qquad b_1 = -j\omega\boldsymbol{E}^+\cdot\boldsymbol{P}_0/2P_n \tag{8}$$

$$a_2 = j\omega\mu_0\boldsymbol{H}^-\cdot\boldsymbol{M}_0/2P_n \qquad b_2 = j\omega\mu_0\boldsymbol{H}^+\cdot\boldsymbol{M}_0/2P_n \tag{9}$$

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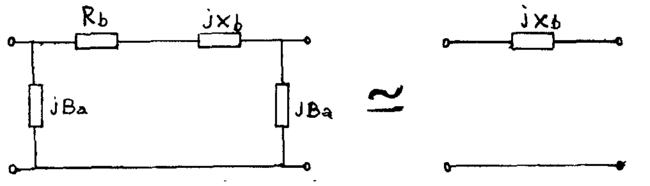

(b)

Fig. 2. Equivalent network representation.

where $\boldsymbol{P}_0$ and $\boldsymbol{M}_0$ are induced electric and magnetic dipole moments, respectively, and are given by

$$\boldsymbol{P}_0 = \epsilon_0 \alpha_e \hat{n}\hat{n} \cdot \boldsymbol{E}_t \tag{10}$$

$$\boldsymbol{M}_0 = \bar{\bar{\alpha}}_m \cdot \boldsymbol{H}_t \tag{11a}$$

$$\bar{\bar{\alpha}}_m = \alpha_{mx} \boldsymbol{a}_x \boldsymbol{a}_x + \alpha_{mz} \boldsymbol{a}_z \boldsymbol{a}_z. \tag{11b}$$

Here $\boldsymbol{a}_x$ and $\boldsymbol{a}_z$ are unit vectors tangential to the aperture and $\hat{n}$ is a unit vector normal to the aperture. Expressions for α_e and $\bar{\bar{\alpha}}_m$, the electric and magnetic polarizabilities respectively of the aperture, are available in the literature [10]. The normalization constant P_n is given by

$$P_n = \iint_S \boldsymbol{E} \times \boldsymbol{H} \cdot \boldsymbol{a}_z \, dS = Y_0. \tag{12}$$

Substituting (2), (3), (4), (10), (11), and (12) in (8) and (9), we obtain

$$a_1 = b_1 = -j\omega\epsilon_0 \alpha_e A_1 / 2Y_0 W_{\text{eff}} h \tag{13}$$

$$a_2 = -b_2 = -j\omega\mu_0 Y_0 \alpha_{mx} A_1 / 2W_{\text{eff}} h. \tag{14}$$

The total electric field due to both electric and magnetic dipoles in microstrip line for $z < 0$ is given by

$$(b_1 + b_2)E^- = j\omega A_1(-\epsilon_0 Z_0 \alpha_e + \mu_0 Y_0 \alpha_{mx})/2W_{\text{eff}} h. \tag{15}$$

The reflection coefficient due to the slot discontinuity is defined as

$$\Gamma = (b_1 + b_2)E^- / A_1 E^+ = j\omega(-\epsilon_0 Z_0 \alpha_e + \mu_0 Y_0 \alpha_{mx})/2W_{\text{eff}} h. \tag{16}$$

The equivalent circuit parameter due to the discontinuity can be expressed in terms of the reflection coefficient as discussed below.

III. Equivalent Circuit Representation

The discontinuity structures under consideration are of the zero-resistance type, in which a continuous upper conductor joins the sides of the discontinuity. Hence the equivalent network of both the transverse slot and the circular aperture can be represented as shown in Fig. 2 [11]. If radiation losses are taken into consideration, the equivalent networks will also possess a resistive component.

A. The Modified Oliner Formulas

For the case under consideration, the scattered fields due to the aperture are sensitive to the H_x, H_z, and E_y components of the unperturbed fields in the microstrip line. Therefore, the expressions for the equivalent circuit parameters will contain the polarizability components α_{mx}, α_{mz}, and α_e in the general case. For the quasi-static case, however, H_z is zero. The small-aperture expressions given by Oliner for the shunt element $B_a(=1/X_a)$ and the series element $B_b(=1/X_b)$ [7] normalized to the characteristic admittance, Y_0, of the microstrip line are

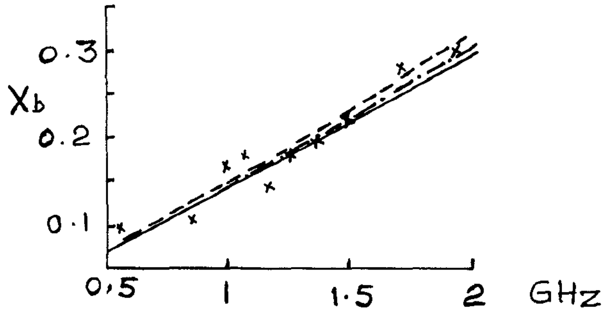

Fig. 3. Variation of normalized series reactance with frequency for thin transverse slot: ——— present theory; —·—·— modified Oliner theory; ---- modified Das theory; × × × × experimental.

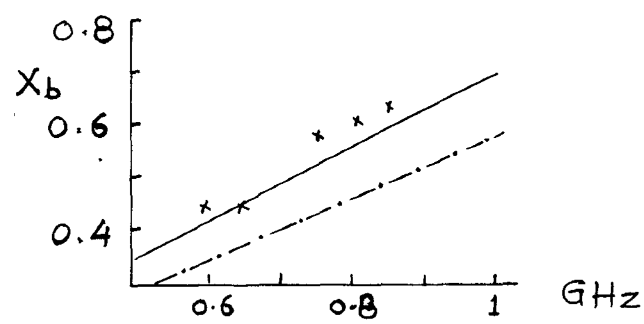

Fig. 4. Variation of normalized series reactance with frequency for circular aperture: ——— present theory; —·—·— modified Oliner's theory; × × × experimental.

modified and are given by

$$B_a = \omega\mu Y_0 \alpha_{mz} h_z h_z^* - \omega \in Z_0 \alpha_e e_y e_y^* \tag{17}$$

$$X_b = -1/B_b = \omega\mu Y_0 \alpha_{mx} h_x h_x^*. \tag{18}$$

The computation shows that the shunt susceptance is negligibly small compared with the series reactance and hence can be neglected as shown in the approximate equivalent circuit in Fig. 2(b). Results for the series reactance due to a thin transverse slot and a circular aperture are shown in Figs. 3 and 4 respectively.

B. The Complex Power Method

The normalized series admittance is given by

$$Y_b = G_b + jB_b = P / Y_0 (\Delta V)^2 \tag{19}$$

where $P = P_r + jP_i$ is the complex radiated power, ΔV is the discontinuity in the modal voltage, and Y_0 is the characteristic wave admittance in microstrip [2].

The discontinuity in the modal voltage, ΔV, is related to the normalized modal vector $\boldsymbol{e}$ and is evaluated following the procedure given in [2]. In the present case the expression for ΔV is obtained as

$$\Delta V = V_m \sum_{n=1,3,\cdots}^{\infty} \frac{\sqrt{2\pi}}{aR} F_n \cos(n\pi\delta/a) 2\sin(n\pi/2)$$
$$\cdot \cosh(n\pi h/a) k[\cos(kl/2) - \cos(n\pi l/2a)] \Bigg/ \left[\left(\frac{n\pi}{a}\right)^2 - k^2\right]. \tag{20}$$

R and F_n are defined in [2], δ is the offset of the transverse slot, a is the width of the ground plane, and $k = (2\pi/\lambda)\sqrt{\epsilon_{r\text{eff}}}$.

With this modification, and the procedure described in [2], the real and imaginary parts of the normalized impedance due to a very thin slot in the center conductor of a microstrip line were evaluated. These results should be divided by 4 because, in the calculation of ΔV, the effect of loading has to be taken into consideration.

The real part of the normalized impedance was found to be negligibly small (0.0020 at 3 GHz). The reactance as a function of frequency is plotted in Fig. 3. However, this approach is applicable only in the case of a very thin slot.

C. The Present Method

The reflection coefficient due to the normalized series reactance (jX_b) is given by

$$\Gamma = Z_n/(2+Z_n) = jX_b/(2+jX_b). \tag{21}$$

When loading due to the discontinuity is small, the reactance can be approximated as

$$jX_b \approx 2\Gamma \tag{22}$$

where Γ is the reflection coefficient computed from (16).

The series reactance was computed for a narrow transverse slot of width 1 mm and length 1.5 cm in a 20 Ω line on an RT Duroid substrate of height 1/16 in. The variation of the normalized series reactance, X_b, is shown in Fig. 3. The series reactance calculated from Oliner's formula (18) using the $\boldsymbol{e}$ and $\boldsymbol{h}$ in microstrip are also shown in the figure for comparison. The results for the case of the circular aperture of diameter 1.5 cm are shown in Fig. 4.

IV. Experimental Verification

In order to verify the theoretical results, a thin slot of width 1 mm and length 1.5 cm was etched in a microstrip line having a characteristic impedance of 20 Ω and a width of 1.58 cm on an RT Duroid substrate of height 1/16 in. The use of a low-impedance line with this substrate was essential. The width of a 50 Ω line is 4.71 mm, and a slot less than 4.7 mm long in this line cannot produce appreciable loading at the input. Thus, the measure of complex reflection and transmission coefficients is extremely difficult in 50 Ω line. For appreciable loading, it was necessary to etch the discontinuity in a low-impedance line which was matched both at the input and output to a standard 50 Ω line with the help of a taper as depicted in Fig. 1.

In order to evaluate the effect of the discontinuity alone, a similar line was fabricated without the discontinuity. The return loss caused by the taper alone was better than 20 dB, showing excellent match. The complex transmission coefficient, T, was measured with an HP sweep oscillator and an HP S-parameter test unit by sweeping the frequency from 0.5 to 2.0 GHz. The calibration was done at a few spot frequencies. In terms of complex transmission coefficient $T=|T|e^{j\theta}$, the required reactance is given by [1]

$$X = -2\sin\theta/|T|. \tag{23}$$

The measured data at spot frequencies for the slot and for a circular aperture of 1.5 cm diameter are shown along with the theoretical results in Fig. 3 and Fig. 4 respectively.

V. Conclusions

Closed-form expressions for the series reactance of two types of discontinuities in the center conductor of microstrip line have been developed. The results based on the quasi-static analysis using equivalent dipole moments are in good agreement with those obtained by modifying the formulation of Oliner [7] and Das [2] and with the experimental data from transmission measurements. Knowledge of the equivalent network parameter is essential in the analysis of periodic structures using such discontinuities.

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Analysis of Stripline Filled with Multiple Dielectric Regions

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Abstract —**The determination of the characteristic impedance of the stripline filled with different dielectric regions is discussed in this paper. Four rectangular dielectric regions whose interfaces are perpendicular to the ground planes are considered. The data on the characteristic impedance and effective dielectric constant are presented for the case of stripline filled with four rectangular regions having different dielectric constants. Results for impedance are compared with the data in the literature. Design data (w/b ratio and effective dielectric constant) are also presented.**

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